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# A Question on the Usefulness of Copyright Protection

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## Abstract

This paper treats the problem of copyright protection. Many works state that strong copyright protection increases producer's surplus and decreases consumer's surplus, but we argue that it is doubtful. We use a repeated game to analyze this problem, and show that weakening copyright protection may improve economic welfare in the sense of Pareto.

## 1 Introduction

The problem finding the optimal level of copyright protection became a serious economic problem. This problem started since several years ago accompanying with the problem of digital copy, especially illegal copy of music files.

To understand this problem, we need to consider why copyright is needed. Firstly, suppose no copyright exists. The cost of copying is usually much cheaper than that of creating, so we can assume this cost is zero. If the original author sells his works, other firms can copy and sell it with cost zero. Because of competition, the price tends to zero, so the original author suffers a big loss. Therefore, he doesn't try to sell his works. Next, suppose copyright exists and is so strong that no firms can copy and sell. Then the original author can gain positive profit. It is not good for economy, however, because the author gains monopolistic profit.

Therefore, too high or too low level of copyright protection is undesirable. So we want to know the optimal level of copyright protection. Many works consider that high level copyright protection increases producer's surplus and decreases consumer's surplus<sup>1</sup>.

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<sup>1</sup>For example, see Landes and Posner(1989).

The aim of this paper is to submit a theoretical counter example on this statement. Some firms allow consumer to try their software as long as he likes. Other firms allow consumer to use their software freely, and depend on the donation from users<sup>2</sup>. The fact these softwares exist and manage as usual tells us an important point. In these examples, author abandons the right to receive money, but his profit is not equal to zero.

Remember our explanation why copyright is needed. We considered the price competition between the author and the copy firms. In usual price competition it is assumed if the price of one firm is not the minimum price, then the sales of his products must be zero, because consumers can buy it from other firms whose price is lower than him. A principle behind this assumption is that; consumers always prefer the cheapest payment. However, the case in the previous paragraph is a counter example.

Existence of this counter example raises a question. Examples show that even if no copyright protection exists, the author can gain positive profit. Can no copyright protection improve economic welfare, in the sense of Pareto? We analyze this problem in the framework of repeated game, and answer this question positively. One example is the case in which consumers' heterogeneity is very large. It is treated in section 2. The other example is the case in which an uncertainty about the value of works exists, treated in section 3.

## 2 The case under large heterogeneity

Consider a repeated game with one supplier and  $n$  consumers.  $\mathcal{N} = \{0, \dots, n\}$  means a player set, and player 0 denotes supplier. Game proceeds below. In the beginning of each period  $t$ , supplier chooses "make" ( $M$ ) or "halt" ( $H$ ). If supplier chooses  $H$ , period  $t$  finishes and every player gains payoff 0. If not, supplier determines a price  $p_t$  from  $[0, \infty)$ <sup>3</sup>. Consumers choose "buy" ( $B$ ), "don't use" ( $N$ ), or "copy" ( $C$ ) simultaneously. The payoff of consumer  $i$  at  $t$  is

$$\begin{aligned} & u_i - p_t && \text{if his action is } B \\ & u_i - Xp_t && \text{if his action is } C \\ & 0 && \text{otherwise} \end{aligned}$$

where the term  $X$  denotes the level of copyright protection. We consider only two different types of  $X$ . One is the case  $X = 0$ , called *no copyright*

<sup>2</sup>Street performance is another example in which the profit of supplier depends on the donation.

<sup>3</sup>We don't assume that supplier determines  $p_t$  from  $\mathbf{R}$ , because total payoff function cannot become well-defined.

*protection* case. Another is the case  $X = 1$ , called *full copyright protection* case. If  $k_t$  consumers choose  $B$  and  $l_t$  consumers choose  $C$ , the payoff of supplier at period  $t$  is  $k_t p_t + X l_t p_t - c$ . The term  $c$  denotes cost to make his works. The total payoff of each player is total sum of payoff with discount rate  $\delta$ , that is, if player  $i$  gains payoff  $x_i^t$  in period  $t$ , his total payoff is

$$\sum_{t=1}^{\infty} \delta^{t-1} x_i^t.$$

The situation this game expresses is here. Supplier can make his works or not. If he chooses to make it, he must pay a fixed cost  $c$ . Suppose that supplier chooses to make his works. If consumer  $i$  buys and consumes it, he gains the payoff  $u_i$  and loses  $p_t$  dollars. In no copyright protection case (that is,  $X = 0$ ), he can copy freely and consume this works. In full copyright protection case ( $X = 1$ ), he cannot copy and must buy works if he wants to consume it. Of course, he can ignore supplier's works.

We can find a general result in the case of no copyright protection.

**theorem 2.1** Suppose  $X = 0$  and  $U = u_1 + \dots + u_n > c$ . For every  $v \in \mathbf{R}_{++}^{n+1}$  such that  $v_i \leq u_i$  for every  $i = 1, \dots, n$  and  $v_0 + \dots + v_n = U - c$ , if  $1 - \delta$  is sufficiently small, then there exists a Nash equilibrium in which each player gains payoff  $\frac{1}{1 - \delta} v_i$ .

**proof** We consider the sequence of actions  $(a_0^t, \dots, a_n^t)_t$  such that if  $t \equiv i \pmod{n}$ , supplier chooses  $a_0^t = \left( M, \frac{1 - \delta^n}{\delta^{i-1}(1 - \delta)} (u_i - v_i) \right)$ , only player  $i$  chooses  $a_i^t = B$ , and other players choose  $a_j^t = C$ . This sequence of actions attains desired payoff. Next, we consider a trigger strategy  $(s_0, \dots, s_n)$  such that in period  $t^*$ , player  $i$  takes  $a_i^{t^*}$  if there was no action differs from  $a_i^t$  for any  $t < t^*$ , and  $H$  (if  $i = 0$ ) or  $N$  ( $i \neq 0$ ) otherwise. By the method similar to the proof of folk theorem, it can be shown that if  $1 - \delta$  is sufficiently small, then  $(s_0, \dots, s_n)$  is a Nash equilibrium. ■

**example 1 (first example copyright protection is not effective)**

Suppose  $n = 2$ ,  $u_1 = 0.9$ ,  $u_2 = 0.2$  and  $c = 1$ . At first, we show that there is no equilibrium in which both consumers can consume the works for every period  $t$  under the case  $X = 1$ . We use reductio ad absurdum. Suppose not. Every player has the minimax value 0, so that, in equilibrium the payoff vector must belong to  $\mathbf{R}_+^3$ . The payoff of player 2 is

$$\sum_{t=1}^{\infty} \delta^{t-1} (0.2 - p_t) \geq 0.$$

So, the payoff of player 0 is

$$\sum_{t=1}^{\infty} \delta^{t-1} (2p_t - 1) < 0,$$

a contradiction.

Therefore, in equilibrium the payoff vector  $U^* = (U_0^*, U_1^*, U_2^*)$  satisfies  $(1 - \delta)(U_0^* + U_1^* + U_2^*) < 0.1$  and  $0 \leq U_i^*$ . So, we can find a vector  $v \in \mathbf{R}^3$  such that  $v_0 + v_1 + v_2 = 0.1$  and  $v_i > (1 - \delta)U_i^* \geq 0$ . Also, we can check  $v_i < u_i$  for  $i = 1, 2$ <sup>4</sup>. Using above theorem, we find an equilibrium under  $X = 0$  such that the payoff vector is  $\frac{1}{1 - \delta}v > U^*$ . Therefore, every equilibrium under  $X = 1$  is Pareto dominated by some equilibrium under  $X = 0$ . ■

Next proposition suggests that above example depends on the fact  $u_1$  largely differs from  $u_2$ .

**proposition 2.1** Suppose  $u = u_1 = u_2 = \dots = u_n > \frac{c}{n}$ . Then, there is an equilibrium such that all consumers can consume the works even if  $X = 1$ .

**proof** the couple of actions  $a = ((M, u), B, \dots, B)$  is a Nash equilibrium of component game. So the couple of strategy such that every player takes action  $a$  regardless of history is a Nash equilibrium of repeated game. ■

Next proposition is the opposite of above.

**proposition 2.2** Suppose  $U = u_1 + \dots + u_n < c$ . Then supplier never chooses  $M$  in equilibrium even if  $X = 0$ .

**proof** Since 0 is the minimax point, the equilibrium payoff vector  $U^* = (U_0^*, \dots, U_n^*)$  must belong to  $\mathbf{R}_+^{n+1}$ . If supplier chooses  $M$  in period  $t$ , then the sum of payoff in period  $t$  must be less than 0. Otherwise, the sum of payoff equal to 0. So if once supplier chooses  $M$ ,  $U_0^* + \dots + U_n^* < 0$ , a contradiction. ■

### 3 The case under some uncertainty

The last proposition of section 2 seems to be obvious. However, if there is some uncertainty about the value of works, we can find a counter example of this proposition. In this section, we consider this example.

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<sup>4</sup>For example,  $v_0 + v_2 \geq 0$  means  $v_1 \leq 0.1 < u_1$ .

The model considered in this section is similar to in previous section, except if supplier chooses to make his works in period  $t$ , the value of works  $x^t = (x_1^t, \dots, x_n^t)_t$  determined randomly. Supplier cannot know  $x^t$  until the end of period  $t$ . Consumer  $i$  can know  $x_i^t$  if he consumes works. In full copyright protection case, consumer cannot consume it except he pays  $p$  dollars. Contrastively, in no copyright protection case, consumer can copy and consume it freely, and determines to buy or not to buy depending on  $x_i^t$ .

The payoff of supplier is the same as previous section. The payoff of consumer is rather changing. If he doesn't choose  $N$ , it is expressed by function  $u_i(x, p^*)$ , where  $x$  denotes the value of works and  $p^*$  the payment. If he chooses  $N$ , then his payoff is 0<sup>5</sup>.

**example 2 (second example copyright protection is not effective)**

Suppose  $n = 1, c = 0.08, u(x, p^*) = -e^{-10(x-p^*)} + 0.17$  and  $x^t$  is independent of  $t$  and has an uniform distribution on  $[0, 1]$ <sup>6</sup>.

Firstly, we consider the case of full copyright protection. Suppose  $W_t$  denotes the sum of expected payoffs at period  $t$ . If supplier chooses  $H$  at period  $t$ , then  $W_t = 0$ . If supplier chooses  $M$  and consumer chooses  $N$ , then  $W_t = -0.08$ . Otherwise,

$$W_t = p - 0.08 - E[e^{-10x+10p} + 0.17] = p + \frac{e^{10p}(e^{-10} - 1)}{10} + 0.09$$

which depends on  $p$ . To maximize  $W$ , it increases to  $\frac{-\log(1 - e^{-10})}{10} - 0.01 < 0$  where  $p = \frac{-\log(1 - e^{-10})}{10}$ . So  $W_t \geq 0$  if and only if supplier chooses  $H$  at period  $t$ . Recall that both players have the minimax value 0. Therefore, in equilibrium payoffs must be larger than 0. It means supplier must not choose  $M$  in equilibrium.

Next, we treat the case of no copyright protection. Consider a pair of actions  $a = (M, \frac{1}{10})$  and  $b = (B(\text{if } x \geq \frac{1}{10}), C(\text{otherwise}))$ <sup>7</sup>. The expected payoff of supplier is  $0.01 > 0$ . The expected payoff of consumer is

$$\frac{1}{10}(e^{-1} - 1) + \frac{e}{10}(e^{-10} - e^{-1}) + 0.17 = \frac{1}{10e} + \frac{1}{10e^9} - 0.03 > 0.$$

Because of the folk theorem, there exists a Nash equilibrium such that players choose action  $(a, b)$  for every period  $t$ . The payoff of both player in such equilibrium must be larger than 0.

<sup>5</sup>Notice that we don't assume that  $u_i(0, 0) = 0$ .

<sup>6</sup>If the number of supplier is truly one, the assumption  $x^t$  is independent of  $t$  is so strong that it is not practical. Therefore, we explain the supplier includes infinitely many people in fact. One supplier chooses to make or not and retires at the end of period.

<sup>7</sup>We can explain the action  $b$ , "use as a trial and buy if he likes it".

In this example, consumer is risk-averse and supplier is risk-neutral. Consumer can impose his risk on supplier in no copyright protection case, but it cannot in full copyright protection case. So the supremum of the sum of expected payoff in no copyright case may become larger than in full copyright case. That is the reason why weakened copyright protection improves economic welfare. ■

This example shows that no copyright protection may be much better than full copyright protection under uncertainty. In example 1, every equilibrium payoff on full copyright protection is dominated by some equilibrium payoff on no copyright protection. On the other hand, in example 2 full copyright protection interrupts supplier makes his works, so that, every equilibrium payoff on full copyright protection must be equal to 0. Therefore, every equilibrium payoff on full copyright protection is dominated by *every* equilibrium payoff on no copyright protection.

## 4 Conclusion

Example 1 treats the case in which consumers' heterogeneity is very large, and shows that weak copyright protection may improve economic welfare. Example 2 deals with the case in which there is some uncertainty on value of works, and shows that strong copyright protection may disable supplier from making his works.

Both examples reveal that strong copyright protection is often not effective in the sense of Pareto. So we should be careful to strengthen the level of copyright protection.

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